Intermediate ALGEBRA

6TH EDITION

Functions and Authentic Applications

Jay LEHMANN

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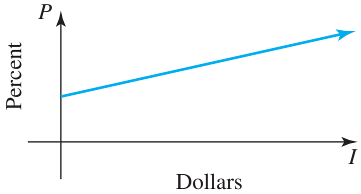

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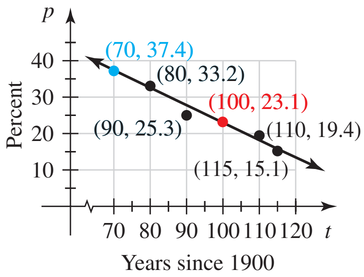

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Year of Birth	Women (years)	Men (years)
1980	77.4	70.0
1985	78.2	71.1
1990	78.8	71.8
1995	78.9	72.5
2000	79.5	74.1
2005	79.9	74.9
2010	81.0	76.2
2014	81.2	76.4

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Average Ticket Prices to Major League Baseball Games (p. 222)

Year	Average Ticket Price (dollars)
1950	1.54
1960	1.96
1970	2.72
1980	4.45
1991	8.84
2000	16.22
2010	26.74
2015	28.94

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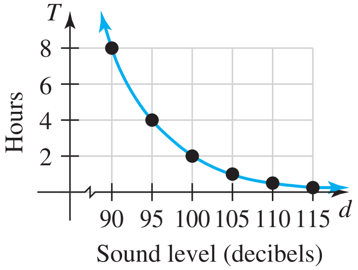

Worldwide Vinyl Record Revenues (pp. 390–391)

Year	Revenue (millions of dollars)
2000	107
2002	64
2004	51
2006	36
2008	66
2010	87
2012	171
2014	347
2015	416

Average Per-Person Consumption of Bottled Water (pp. 449–450)

Year	Average Consumption (gallons per person)
2009	27.6
2010	28.3
2011	29.2
2012	30.8
2013	32.0
2014	34.1
2015	36.5

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Year	Number of Internet Users (millions)
2003	179.5
2006	206.0
2009	218.1
2012	249.6
2015	283.7

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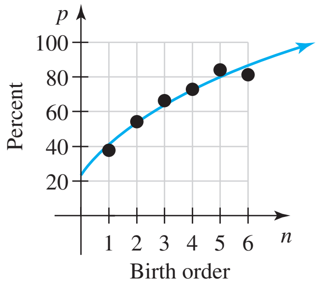

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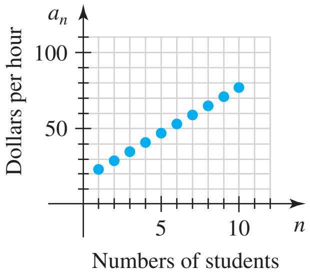

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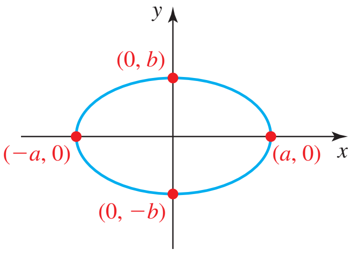

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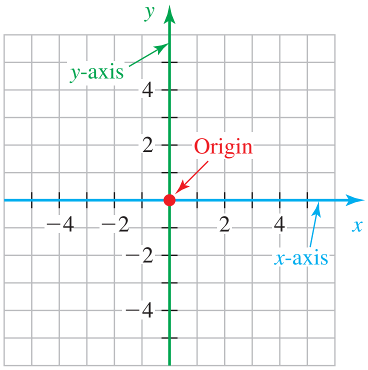

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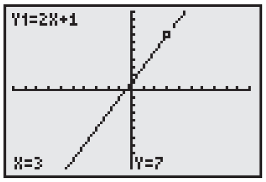

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Preface

"The question of common sense is always, 'What is it good for?'—a question which would abolish the rose and be answered triumphantly by the cabbage."

—James Russell Lowell

These words seem to suggest that poet and editor James Russell Lowell (1819–1891) took Intermediate Algebra. How many times have your students asked, "What is it good for?" After years of responding "You'll find out in the next course," I began an ongoing quest to develop a more satisfying and substantial response to my students' query.

This ongoing quest has led me to author three algebra texts and, most recently, a new Prestatistics text, *A Pathway to Introductory Statistics*. I have a passion for using authentic data, centered around a curve-fitting approach to help students learn in context.

Curve-Fitting Approach Although there are many ways to center an Intermediate Algebra course around authentic applications, I chose a curve-fitting approach for several reasons. A curve-fitting approach

- allows great flexibility in choosing interesting, authentic, current situations to model.
- emphasizes concepts related to functions in a natural, substantial way.
- deepens students' understanding of functions because it requires students to describe functions graphically, numerically, symbolically, and verbally.
- unifies the many diverse topics of a typical Intermediate Algebra course.

There is yet one more reason I chose a curve-fitting approach. Intermediate Algebra is meant to prepare some students for a Calculus STEM track and others for Statistics, Liberal Arts Math, and so on (non-STEM tracks). This is a great challenge because Calculus, Statistics, and so on are vastly different courses not only in content but also in the type of problem solving they require. Teaching algebra with curve fitting empowers instructors to prepare students for all types of content and problem solving.

To fit a curve to data, students learn the following four-step modeling process:

1. Examine the data set to determine which type of model, if any, to use.
2. Find an equation of the model.
3. Verify that the model fits the data.
4. Use the model to make estimates and predictions.

This four-step process weaves together topics that are crucial to the course. Students must notice numerical patterns from data displayed in tables, recognize graphical patterns in scatterplots, find equations of functions, graph and evaluate functions, and solve equations.

Not only does curve fitting foster cohesiveness within chapters, but it also creates a parallel theme for each chapter that introduces and discusses a new function. This structure enhances students' abilities to observe similarities and differences among fundamental functions such as linear functions, exponential functions, logarithmic functions, quadratic functions, rational functions, and radical functions.

Curve fitting serves as a portal for students to see the usefulness of mathematics so they become fully engaged in the class. Once involved, students are more receptive to all aspects of the course.

NEW TO THE SIXTH EDITION

Students will benefit from the following changes to the sixth edition of *Intermediate Algebra: Functions and Authentic Applications*:

- In previous editions, all authentic data sets in the print text were represented by similar, yet generic (inauthentic), data sets in MyLab Math to provide algorithmically generated similar exercises for students completing homework in MyLab Math. However, in the new edition, where possible, MyLab Math exercises maintain the authenticity of the data. This has been accomplished by sampling from a large data set to generate six authentic data sets that inherit the same trend.
- ***MyLab Math Exercises:*** The number of skill, modeling, and conceptual exercises in MyLab Math has been increased to fully capture the spirit of the print textbook. In fact, for the first time ever, Related Review exercises (described later in the preface) will be assignable in MyLab Math.
- ***Large Data Sets:*** Many students who use this textbook will not perform regression analysis in their careers, but some *will* work with large data sets. Such work will also help prepare students to take Statistics. With this in mind, new exercises that involve large data sets have been sprinkled throughout the textbook. They directly follow the heading "Large Data Sets." The data sets consist of as many as thousands of rows and tens of columns of data.
- ***Downloadable Data Sets:*** To support the appropriate use of technology when completing exercises and labs, data sets that consist of 16 or more data values can now be downloaded as Excel files at MyLab Math and at the Pearson Downloadable Student Resources for Math and Statistics website: http://www.pearsonhighered.com/mathstatsresources. These data sets in MyLab Math can also be opened in StatCrunch. Exercises that involve such data sets are flagged in the print textbook by the icon .
- ***Augmented Data Sets:*** To make the data sets as current and relevant as possible, 172 data sets in examples and exercises have been augmented to include values for recent years.
- ***New Data Sets:*** 150 data sets in examples and exercises have been replaced with more compelling and contemporary topics such as immigration, legalization of marijuana, and trust in newspapers.
- ***Climate Change Labs:*** All five Climate Change labs have been updated to address the latest data and political events concerning this incredibly important global issue.
- ***Graphing Calculator Instructions:*** Appendix B, which consists of TI-83/TI-84 graphing calculator instructions, was available only online in the previous edition. To make the appendix more accessible to students, it is now included in the textbook.
- ***StatCrunch Instructions:*** Some departments that require StatCrunch for their Statistics courses introduce StatCrunch in their Intermediate Algebra courses. To support such departments, Appendix C, which contains StatCrunch instructions, has been added to the textbook.
- ***Section Opener Explorations:*** Explorations that can be used at the start of a section have been moved from the preceding section to the current section. The new placement will visually remind instructors to assign such explorations and make it easier for students to access them.
- ***Statistics Terminology:*** To better support students who will take Statistics, the terminology has been improved: The words *scattergram*, *independent variable*, and *dependent variable* have been replaced with *scatterplot*, *explanatory variable*, and *response variable*.

- ***Logarithm Preparation:*** The technique of converting expressions in exponential form to and from expressions in radical form has been added to Section 4.2 to better lay a foundation for logarithms in Chapter 5.
- ***Color:*** More color has been used to enhance connections between equations, graphs, tables, and coordinates of ordered pairs.

CONTINUED FROM THE FIFTH EDITION

Unique Organization Some students find it hard to stay interested because they've "seen it all before" in Elementary Algebra. To address this issue, content that will be new to most students is presented in Sections 1.1, 1.4, and 1.6, as well as in most of Chapters 2–11. Section 1.1 sets the tone that this course will be different, interesting, alive, and relevant, inviting students' creativity into the classroom.

Early Functions Although some textbooks introduce functions early in the course, the concept is rarely included in subsequent sections, and when it is included, the treatment is light. In this textbook, functions are introduced early (Section 1.6) and are emphasized throughout the book in meaningful ways such as by curve fitting, providing students with a solid foundation for subsequent courses such as Trigonometry, College Algebra, and Precalculus.

Early Logarithmic Functions Unlike the organization in most textbooks, exponential functions and logarithmic functions are presented before polynomial functions, rational functions, and radical functions. The coverage of exponential functions directly follows that of linear functions so students can see the dual nature of these two functions (by comparing the slope addition property with the base multiplier property). Professors who have used the preceding editions have commented over and over again how much they value an early-logarithm organization. Although rational functions and radical functions present their own challenges, most students have more difficulties with logarithmic functions, and it pays to have them learning about this concept while they still have energy.

Modeling Exercises To give this sixth edition a current and lively feel, the vast majority of the hundreds of modeling exercises in the text have been updated or replaced. Most of the application exercises contain tables of data, but some describe data in paragraph form to give students practice in picking out relevant information and defining variables. Both types of applications are excellent preparation for subsequent courses (especially Statistics).

Group Explorations All sections of this text contain one to three explorations that support student investigation of a concept. Instructors can use explorations as collaborative activities during class time or as part of homework assignments. The "Section Opener" explorations are meant to have students discover the section's concepts at the start of class. The other explorations are designed to have students apply concepts they have learned in the section in new ways. Both types of explorations can empower students to become active explorers of mathematics and open the door to the wonder and beauty of the subject.

Taking It to the Lab Sections Laboratory assignments have been included at the end of most chapters to deepen students' understanding of concepts and the scientific method. These labs reinforce the idea that mathematics is useful. They are also an excellent avenue for more in-depth writing assignments.

Some of the labs are about climate change and have been written at a higher reading level than the rest of the text in order to give students a sense of what it is like to perform research. Students will find that by carefully reading (and possibly rereading) the background information, they can comprehend the information and apply concepts they have learned in the course to make estimates and predictions about this compelling, current, and authentic situation.

Balanced Extensive Homework Sections Most exercise sets contain a large number of modeling, skill, and conceptual exercises to allow professors maximum flexibility in setting assignments.

Related Review These exercises (in every section of Chapters 4–11) relate current concepts to previously learned concepts. Such exercises assist students in seeing the "big picture" of the course. This exercise type is now also assignable in MyLab Math.

Expressions, Equations, Functions, and Graphs These exercises (in every section of Chapters 4–11) help students gain a solid understanding of those core concepts, including how to distinguish among them.

Technology The text assumes students have access to technology such as the TI-83 or TI-84 graphing calculator, Excel, or StatCrunch. Technology of this sort allows students to construct scatterplots and check the fit of a model quickly and accurately. It also empowers students to verify their results from Homework exercises and efficiently explore mathematical concepts in the Group Explorations.

The text supports instructors in holding students accountable for all aspects of the course without the aid of technology, including finding equations of models. (Regression equations are included in the Answers section because it can be difficult or impossible to anticipate which points a student will choose in trying to find a reasonable equation.)

Appendix A: Reviewing Prerequisite Material Appendix A can be used to remind students of important topics typically addressed in an Elementary Algebra course. Examples and exercises are included in each section.

Appendix B: Using a TI-83 or TI-84 Graphing Calculator Appendix B contains step-by-step instructions for using the TI-83 and TI-84 graphing calculators. A subset of this appendix can serve as a tutorial early in the course. In addition, when the text requires a new calculator skill, students are referred to the appropriate section in Appendix B.

Appendix C: Using StatCrunch Appendix C contains step-by-step instructions for using StatCrunch. The appendix describes how to enter data, construct scatterplots, and find regression equations.

Exposition If students can't make sense of the prose, it doesn't matter how precise it is. One of my top goals is to write descriptions that are straightforward, accessible, clear, and rigorous.

Tips for Success Many sections close with tips that are intended to help students succeed in the course. A complete listing of these tips is included in the Index.

Additional Topics Chapter Topics typically taught in Intermediate Algebra that cannot be connected with a curve-fitting approach at the appropriate level are assembled in Chapter 11. Each section contains a Section Quiz feature. The union of these quizzes can be used as a set of review exercises for Chapter 11. Instructors who wish to "cut and paste" sections from that chapter into earlier chapters can append these quizzes to the appropriate Chapter Review exercises.

GETTING IN TOUCH

I would love to hear from you and would greatly appreciate receiving your comments regarding this text. If you have any questions, please ask them, and I will respond.

Thank you for your interest in preserving the rose.

Jay Lehmann
MathNerdJay@aol.com

Pearson MyLab

Resources for Success

Get the Most Out of MyLab Math

for *Intermediate Algebra*, Sixth Edition, by Jay Lehmann

When it comes to developmental math, one size does not fit all. Jay Lehmann's *Intermediate Algebra* offers market-leading content written by an author-educator, tightly integrated with the #1 choice in digital learning—MyLab Math. MyLab Math courses can be tailored to the needs of instructors and students, while weaving the author's voice and unique approach into all elements of the course. Learning mathematical concepts through authentic data comes through from the text to the MyLab course seamlessly.

Take advantage of the following resources to get the most out of your MyLab Math course.

Conceptual Understanding and Motivation

New! Large Data Sets in exercises and explorations get students accustomed to working with as many as ten thousand rows of data.

Data sets that involve 16 or more values are available for download to support the appropriate use of technology. Noted with a DATA icon, these exercises are ideal for using technology, like StatCrunch or Excel, to analyze the data and synthesize concepts. In today's age of "big data," it's important for students to see how technology can efficiently and accurately help when working with large data sets.

Large Data Sets

53. DATA Access the data about airborne times and distances of Delta Airlines flights, which are available at MyLab Math and at the Pearson Downloadable Student Resources for Math & Stats website. Let T be the airborne time (in minutes) and D be the distance (in miles) for a flight.

a. Construct a scatterplot of the data.

b. Give a possible reason why the scatterplot consists of vertically aligned clumps of data points.

c. On the basis of just the scatterplot, guess whether Delta offers more *routes* that are less than 1000 miles or greater than 1000 miles. On the basis of just the scatterplot, why is it not possible to be sure?

d. Print your scatterplot and draw a linear model.

e. Estimate the slope of the linear model. What does it mean in this situation?

Delta_Airborne_Times_and_Distances.txt

StatCrunch | Applets | Edit | Data | Stat | Graph | Help

Row	Destination	Airbone Tim	Distance (m
1	ABQ	179	1267
2	ABQ	179	1267
3	ABQ	183	1267
4	ABQ	172	1267
5	ABQ	181	1267
6	ABQ	184	1267
7	ABQ	175	1267
8	ABQ	179	1267
9	ABQ	183	1267
10	ABQ	173	1267
11	ABQ	179	1267
12	ABQ	182	1267
13	ABQ	172	1267
14	ABQ	173	1267
15	ABQ	182	1267
16	ABQ	159	1267
17	ABQ	164	1267
18	ABQ	166	1267
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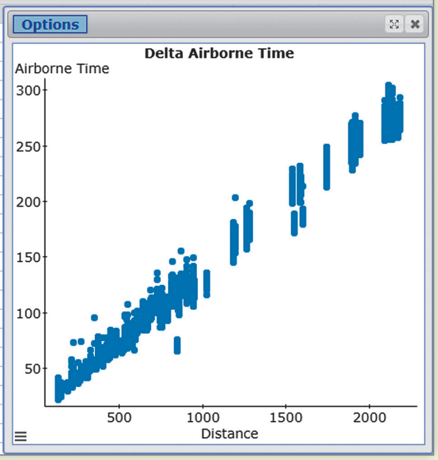
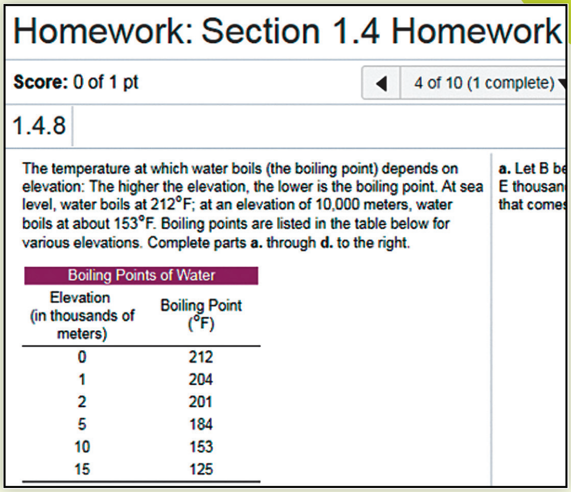
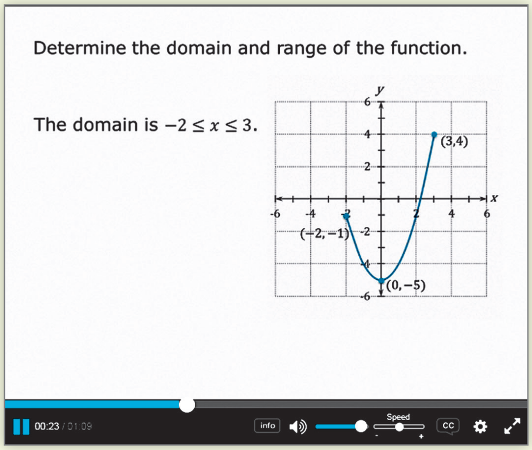
21	ABQ	177	1267
22	ABQ	162	1267
23	ABQ	171	1267



New! StatCrunch is a web-based statistical software available from within the MyLab Math course that students can use to easily analyze data sets from exercises and the text. Through StatCrunch users can access tens of thousands of shared data sets, create and conduct online surveys, perform complex analyses using the powerful statistical software, and generate compelling reports.

New! Select exercises with authentic data have been carefully revised to retain authentic data values, even when regenerating algorithmically. Oftentimes students sacrifice working with real-world data when they regenerate exercises with new values in MyLab Math. In this revision, the author has taken special care to ensure that many exercises' algorithmic versions of the question still ask the student to work with actual data pulled from real-world situations.

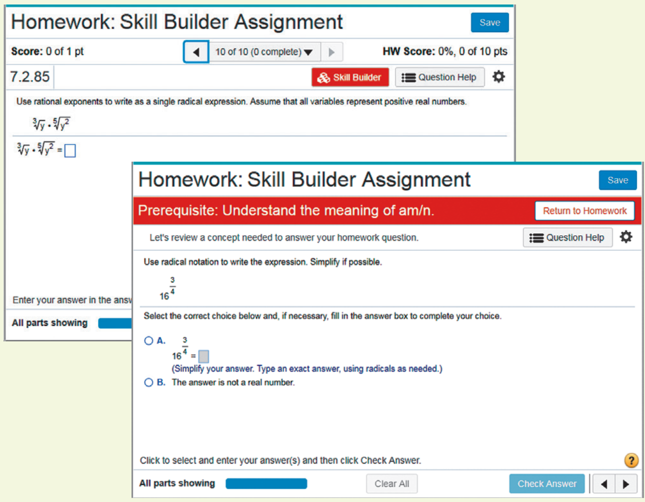




Updated! The video program provides students with extra help for each objective of the textbook. The videos highlight key examples, and a modern interface allows easy navigation. Videos have been updated to reflect all changes in the current edition.

Personalized Learning and Preparedness

New! Skill Builder exercises offer just-in-time additional adaptive practice. The adaptive engine tracks student performance and delivers questions to each individual that adapt to his or her level of understanding. This new feature allows instructors to assign fewer questions for homework, allowing students to complete as many or as few questions needed.



Enhanced Sample Assignments make course set-up easier by giving instructors a starting point for each section and chapter. Homework assignments have been carefully curated for this specific text and include a thoughtful mix of question types. Find these sample assignments in the Assignment Manager, under Copy and Assign Sample Assignments.

pearson.com/mylab/math

Resources for Success

Pearson MyLab

Instructor Resources

The following instructor resources are available to download from the Instructor Resource Center at www.pearson.com*, or in your MyLab Math course.*

Instructor's Resource Manual

This manual, written by the author, contains suggestions for pacing the course and creating homework assignments. It discusses how to incorporate technology and how to structure project assignments. The manual also contains section-by-section suggestions for presenting lectures and for undertaking the explorations in the text.

Power Points

These fully editable lecture slides include definitions, key concepts, and examples for use in a lecture setting and are available for each section of the text.

Instructor's Solutions Manual

This manual includes complete solutions to the even-numbered exercises in the text.

TestGen

TestGen enables instructors to build, edit, print, and administer tests by using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple, but equivalent, versions of the same question or test with the click of a button. Instructors can also modify test-bank questions or add new questions. Tests can be printed or administered online. The software and test bank are available for download from Pearson's online catalogue.

Student Resources

New! Concepts and Explorations Notebook: Working with Authentic Data

This new compelling resource for students correlates to the text and provides students with opportunities to dig into data and solve problems using pencil and paper. The workbook includes:

- Explorations that offer collaborative activities to support discovery of key concepts.
- Modeling exercises with authentic data that give students more practice on this multifaceted concept, that can be sometimes hard to fully accomplish through MyLab Math.
- Projects that can be either open-ended or more guided, and ask students to dig deeper into a data set and think critically.
- Graphing exercises that ask students to practice graphing on their own, beyond what they do in MyLab Math.
- Mini-Essay questions that prompt students to think conceptually, also beyond what they do in MyLab Math!

Student's Solutions Manual

This manual contains the complete solutions to the odd-numbered exercises in the Homework sections of the text.

To the Student

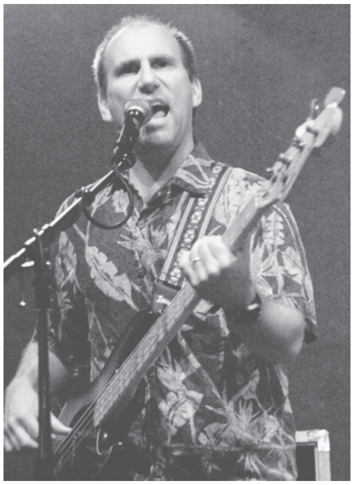
You are about to embark on an exciting journey. In this course, you will learn not only more about algebra but also how to apply algebra to describe and make predictions about authentic situations. "Authentic situations" might make you think twice, but this just means situations that are *really* happening in the world. This text contains data that describe hundreds of these situations. Most of the data have been collected from recent publications, so, the information is current and of interest to the general public. There is data about wearable devices, success in school, climate change, sports, and so on. I hope it interests you too.

Working with authentic data will make mathematics more meaningful. While working with data about authentic situations, you will learn mathematical concepts that will be easier to learn because they will be connected to familiar contexts. And you will see that almost any situation can be viewed mathematically. That vision will help you understand the situation and make estimates and/or predictions.

Many of the problems you will explore in this course involve data collected in a scientific experiment, survey, or census. The practical way to deal with such data sets is to use technology. So, a graphing calculator or computer system is required.

Analyzing authentic situations is a lifelong skill. We are living in the "age of data." In addition to working with data sets in this text, your instructor may assign some of the labs. Here you will collect data through experiment or research. This will give you a more complete picture of how you can use the approaches presented in this text in everyday life, and likely in your lifelong careers. Being able to work with and understand data can lead to higher-paying jobs and success.

Hands-on explorations are rewarding and fun. This text contains explorations with step-by-step instructions that will lead you to *discover* concepts, rather than hear or read about them. Because discovering a concept is exciting, it is more likely to leave a lasting impression on you. Also, as you progress through the explorations, your ability to make intuitive leaps will improve, as will your confidence in doing mathematics. Over the years, students have remarked to me time and time again that they never dreamed that learning math could be so much fun.



Jay has a wide variety of interests. He is pictured here playing with his rock band, The Procrastinistas. (Photo courtesy of Rick Gilbert)

This text contains special features to help you succeed. Many sections contain a Tips for Success feature. These tips are meant to inspire you to try new strategies to help you succeed in this course and future courses. If you browse through all the tips early in the course, you can take advantage of as many of them as you wish. Then, as you progress through the text, you'll be reminded of your favorite strategies. A complete listing of Tips for Success is included in the Subject Index.

Other special features that can support you include Warnings, which can help you avoid common misunderstandings; Key Points summaries, which can help you review and retain concepts and skills addressed in the chapter you have just read; Related Review exercises, which can help you understand current concepts in the context of previously learned concepts; and Expressions, Equations, Functions, and Graphs exercises, which can help you understand and distinguish among these four core concepts.

Feel free to contact me. It is my pleasure to read and respond to e-mails from students who are using my text. If you have any questions or comments about the text, feel free to contact me.

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Acknowledgments

Writing a modeling text is an endurance run I couldn't have completed without the dedicated assistance of many people. First, I'm greatly indebted to Keri, my wife, who yet again served as an irreplaceable sounding board for the countless decisions that went into creating this book.

I acknowledge several people at Pearson Education. I'm very grateful to Editor-in-Chief Michael Hirsch, who has shared in my vision for this text and made significant investments to make that vision happen. The book has been greatly enhanced through the support of Senior Acquisitions Editor Rachel Ross, who made a multitude of contributions, including assembling an incredible team to develop and produce this text. The team includes Content Producer Tamela Ambush, who handled countless tasks to support me in preparing the manuscript for production, leading to a significantly better book.

Heartfelt thanks goes to Project Managers Thomas Russell and Erin Hernandez, who both orchestrated the many aspects of production.

And thanks to Rick Gilbert for the awesome photograph in the "To the Student" section of the author performing with his band, The Procrastinistas, at the Hotel Utah in San Francisco.

I thank these reviewers, whose thoughtful, detailed comments helped me sculpt this text into its current form:

Scott Adamson, *Chandler-Gilbert Community College*
Thomas Adamson, *Phoenix College*
Ken Anderson, *Chemeketa Community College*
Gwen Autin, *Southeastern Louisiana University*
Mona Baarson, *Jackson Community College*
Sam Bazzi, *Henry Ford Community College*
Joel Berman, *Valencia Community College—East*
Nancy Brien, *Middle Tennessee State University*
Ronnie Brown, *University of Baltimore*
Barbara Burke, *Hawaii Pacific University*
Laurie Burton, *Western Oregon University*
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R. Messer, *American River College*
Jason L. Miner, *Santa Barbara City College*
Nolan Mitchell, *Chemeketa Community College*
Camille Moreno, *Cosumnes River College*
Lisa Mosser, *Jackson Community College*
Ellen Musen, *Brookdale Community College*
Charlie Naffziger, *Central Oregon Community College*
Chris Nord, *Chemeketa Community College*
Donna Marie Norman, *Jefferson Community College*
Denise Nunley, *Glendale Community College*
Karen D. Pain, *Palm Beach State College*
Ernest Palmer, *Grand Valley State University*
Ellen Rebold, *Brookdale Community College*
Jody Rooney, *Jackson Community College*
James Ryan, *State Center Community College District, Clovis*
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Index of Applications

F

Linear Equations and Linear Functions

1

Do you use wearable devices such as fitness trackers and smartwatches? The percentages of Americans who use wearable devices are shown in Table 1 for various age groups. In Exercise 9 of Homework 2.1, you will estimate the age at which 13% of Americans use wearable devices.

Table 1 Percentages of Americans Who Use Wearable Devices

Age Group (years)	Age Used to Represent Age Group (years)	Percent
25–34	29.5	23
35–44	39.5	22
45–54	49.5	15
55–64	59.5	11
over 65	70.0	6

Source: *Gallup*

In this course, you will use mathematics to describe many authentic situations, such as the wearable-device data. You will use these descriptions to make estimates and predictions, as in estimating the age at which 13% of Americans use wearable devices. You will also estimate by how much the number of Internet users worldwide is increasing per year, you will predict in which year there will be 1.0 thousand collisions at highway–railroad crossings, and you will predict the percentage of births outside marriage in the United States in 2021.

A major objective of this text is to help you view the world in a mathematical manner. That viewpoint will allow you to recognize important patterns—patterns that will enable you to make estimates and predictions like the ones just mentioned.

In this chapter, we will discuss how to describe a line by using a *graph*, an *equation*, and a *table*. We will also discuss how to describe the steepness of a line. Finally, we will work with an important group of lines represented by *linear functions*. We will lay the groundwork so that in Chapter 2 we can use lines to describe authentic situations.

1.1 Using Qualitative Graphs to Describe Situations

Objectives

- » Describe the meaning of qualitative graphs.
- » Identify explanatory variables and response variables.
- » Describe the meaning of an *intercept* of a curve.
- » Sketch qualitative graphs.
- » Identify increasing curves and decreasing curves.
- » Describe a concept or procedure.

In this section, we will use qualitative graphs to describe authentic situations. A **qualitative graph** is a graph without scaling (tick marks and their numbers) on the axes.

Describe the Meaning of Qualitative Graphs

How can we make sense of a qualitative graph?

Example 1 Describe the Meaning of a Qualitative Graph

For children whose parents earn I dollars per years, let P be the percentage of the children who attend college. The qualitative graph displayed in Fig. 1 describes the relationship between I and P (Source: *National Center for Education Statistics*). What does the graph tell us?

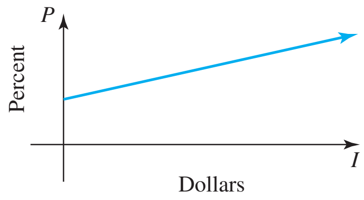

Figure 1 Parents' income and percentage of children who attend college

Solution

The graph (or curve) tells us that the higher the income of parents, the greater the percentage of children who attend college will be.

A curve is said to be *linear* if it forms a straight line. The curve in Fig. 1 is linear.

Explanatory and Response Variables

In Example 1, we concluded that the higher the income of parents, the greater the percentage of children who attend college will be. Because I affects (explains) P, we call I the *explanatory variable*. We call P the *response variable* because P is affected by (responds to) I.

Definition Explanatory and response variables

Assume that an authentic situation can be described by using the variables t and p, and assume that t affects (explains) p. Then

- We call t the **explanatory variable** (or **independent variable**).
- We call p the **response variable** (or **dependent variable**).

Example 2 Identifying Explanatory and Response Variables

For each situation, identify the explanatory variable and the response variable:

1. You are waiting in line to go to a concert. Let T be the number of minutes you must wait, and let N be the number of people ahead of you when you first get in line.
2. Let n be the number of times a person can lift dumbbells that weigh w pounds.

Solution

1. The more people ahead of you when you first get in line, the more time you must wait. The number of people ahead of you, N, affects (explains) your wait time, T. So, N is the explanatory variable and T is the response variable. (Your wait time does *not* affect the number of people ahead of you.)
2. The heavier the dumbbells, the fewer times the person can lift them. The dumbbells' weight, w, affects (explains) the number of times the person can lift them, n. So, w is the explanatory variable and n is the response variable. (The number of times the person can lift the dumbbells does *not* affect the dumbbells' weight.)

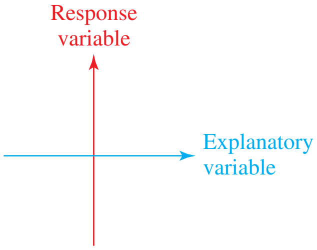

Figure 2 Match the horizontal axis with the explanatory variable and the vertical axis with the response variable

For graphs, we describe the values of the explanatory variable along the horizontal axis and the values of the response variable along the vertical axis (see Fig. 2). For example, in Fig. 1, we describe the values of the explanatory variable I along the horizontal axis and we describe the values of the response variable P along the vertical axis.

Example 3 Describe the Meaning of a Qualitative Graph

Let A be the average age (in years) when men first marry, and let t be the number of years since 1900. (For example, $t = 1$ represents the year 1901.) In Fig. 3, the graph describes the relationship between the variables t and A. What does the graph tell us?

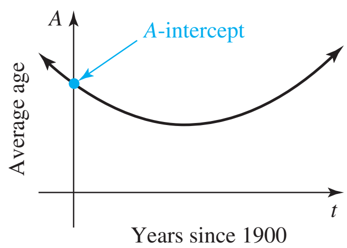

Figure 3 The average age when men first marry

Solution

The graph tells us that the average age when men first marry decreased each year for a while and then increased each year after that.

We say the curve sketched in Fig. 3 is a *parabola*.

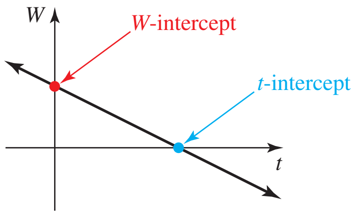

Figure 4 Intercepts of a line

Intercepts of a Curve

In Fig. 3, note that the curve and the A-axis intersect. The point of intersection is an *A-intercept*. An **intercept** of a curve is any point where the curve and an axis (or axes) intersect. Two more examples of intercepts are shown in Fig. 4 for a linear curve.

Sketching Qualitative Graphs

In Examples 4–6, we sketch qualitative graphs that describe given situations.

Example 4 Sketching a Qualitative Graph

Let C be the cost (in dollars) of a 30-second ad during the Super Bowl at t years since 1987. For most years, the annual increase in cost is more than the previous annual increase in cost. Sketch a qualitative graph that describes the relationship between C and t.

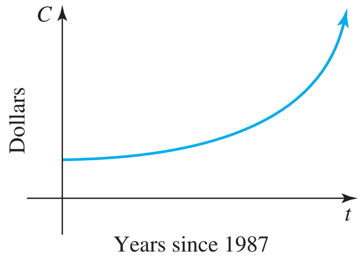

Figure 5 Cost of Super Bowl ads

Solution

Because the year affects (explains) the cost of an ad, t is the explanatory variable and C is the response variable. So, we let the horizontal axis be the t-axis and the vertical axis be the C-axis (see Fig. 5). Because ads were not free in 1987 $(t = 0)$, the C-intercept is above the origin. The costs are increasing, so we sketch a curve that goes upward from left to right. Because most increases are more than the previous increase, the curve should "bend" upward from left to right.

Some *exponential* curves have shapes similar to the shape of the curve sketched in Fig. 5.

Increasing and Decreasing Curves

If a curve goes upward from left to right, we say it is an **increasing curve** (see Fig. 6). For example, the cost curve in Fig. 5 is increasing. If a curve goes downward from left to right, we say it is a **decreasing curve** (see Fig. 7).

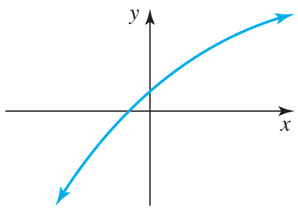

Figure 6 Increasing curve

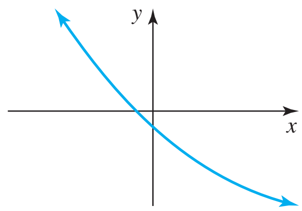

Figure 7 Decreasing curve

In this chapter and future chapters, we will discuss the curves mentioned in this section more thoroughly and use them to make predictions—linear curves in this chapter and Chapter 2, exponential curves in Chapters 4 and 5, and quadratic curves in Chapters 6 and 7. In Chapter 3, you will make predictions with two linear curves.

Example 5 Sketching a Qualitative Graph

Hot coffee is poured into a cup at room temperature. Let F be the temperature (in degrees Fahrenheit) of the coffee at t minutes since the coffee was poured. Sketch a qualitative graph that describes the relationship between the variables t and F.

Solution

Note that t affects (explains) F, so we let the horizontal axis be the t-axis and the vertical axis be the F-axis (see Fig. 8). Because the coffee cools with time, the curve should be decreasing. Further, the curve should show that the drop in temperature during any minute is less than the drop in temperature in the previous minute. (Why?)

The coffee's temperature will not go below room temperature, so the curve should eventually level off.

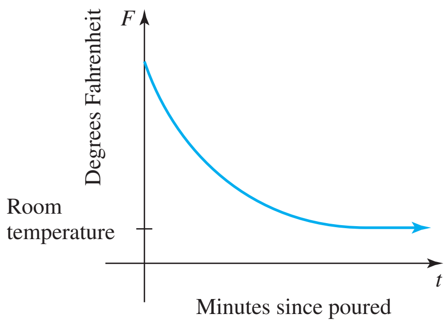

Figure 8 Temperature of a cup of coffee

In Examples 4 and 5, the explanatory variable represents time. Let's explore a situation in which the explanatory variable stands for something else.

Example 6 Sketching a Qualitative Graph

Suppose the latest Radiohead album is about to be released. Let n be the number of albums that will be sold if a dollars are spent on advertising. Sketch a qualitative graph that describes the relationship between the variables a and n.

Solution

The amount of money spent on advertising affects (explains) the number of albums sold, so we let the horizontal axis be the a-axis and the vertical axis be the n-axis (see Fig. 9). Because both n and a must be nonnegative (why?), the qualitative curve is in *quadrant I* (and one point of it is on the n-axis). The four quadrants are shown in Fig. 10.

Even if no money is spent on advertising, some albums will be sold. So the n-intercept should be above the origin. The more money spent on advertising, the greater the sales, so the curve should be increasing. There are only so many people, however, who would buy the album no matter how much advertising is done, so the curve should level off.

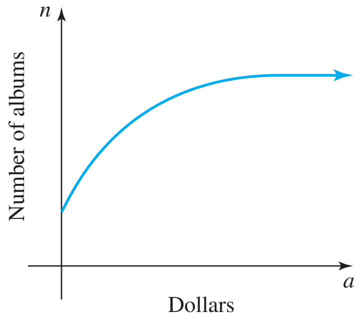

Figure 9 Spending on advertising and albums sold

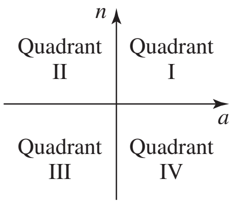

Figure 10 The four quadrants

Describing a Concept or Procedure

In some homework exercises, you will describe in general a concept or procedure.

Guidelines on Writing a Good Response

- Create an example that illustrates the concept or outlines the procedure. Looking at examples or exercises may jump-start you into creating your own example.
- Using complete sentences and correct terminology, describe the key ideas or steps for your example. You can review the text for ideas, but write your description in your own words.
- Describe also the concept or the procedure in general, without referring to your example. It may help to reflect on several examples and what they all have in common.
- In some cases, it will be helpful to point out the similarities and the differences between the concept or the procedure you are describing and other concepts or procedures.
- Describe the benefits of knowing the concept or the procedure.
- If you have described the steps in a procedure, explain why it's permissible to follow these steps.
- Clarify any common misunderstandings about the concept, or discuss how to avoid making common mistakes when following the procedure.

Example 7 Responding to a General Question about a Concept

Describe the meaning of *explanatory variable* and *response variable*.

Solution

Assume that an authentic situation can be described by using the variables t and a and that t affects (explains) a. Then t is the explanatory variable and a is the response variable.

For example, let a be the amount of money (in dollars) that a person is paid for working t hours at a gasoline station. Then t is the explanatory variable and a is the response variable because the number of hours worked affects (explains) the pay.

For graphs, we use the horizontal axis to describe values of the explanatory variable and the vertical axis to describe values of the response variable.

Group Exploration

Sketching a qualitative graph

A bathtub is filled with water, and then the plug is pulled out. Let V be the volume of water (in gallons) in the tub at t seconds after the plug is pulled out.

1. Which variable is the explanatory variable? The response variable? Explain.
2. Sketch a qualitative graph that describes the relationship between V and t. Explain.

Taking It One Step Further

3. Carefully describe an experiment you could run to verify the shape of your curve from Problem 2. In particular, explain how you could measure the volume of water at various times. Ask your instructor if you should run such an experiment.

Homework 1.1

For extra help ▶ **MyLab Math** Watch the videos in MyLab Math

1. The deer population in a forest is described during the years between 2010 and the present. Let p be the deer population in the forest and t be the number of years since 2010. Match each graph in Fig. 11 with each scenario. The population
   a. decreased steadily.
   b. increased steadily.
   c. remained steady.
   d. decreased for a while and then increased.

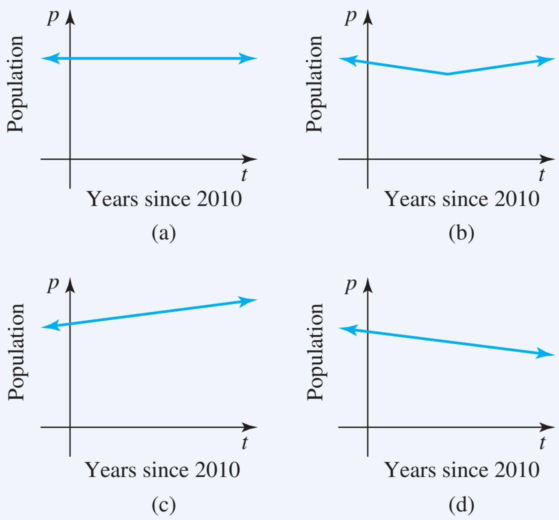

Figure 11 Exercise 1

2. Let A be the amount of rain (in inches) that has fallen in t hours. Match each graph in Fig. 12 with each scenario. The rain fell
   a. harder and harder.
   b. softly and then stopped. After a while, it began raining hard.
   c. hard and then stopped. After a while, it began raining softly.
   d. more and more softly.

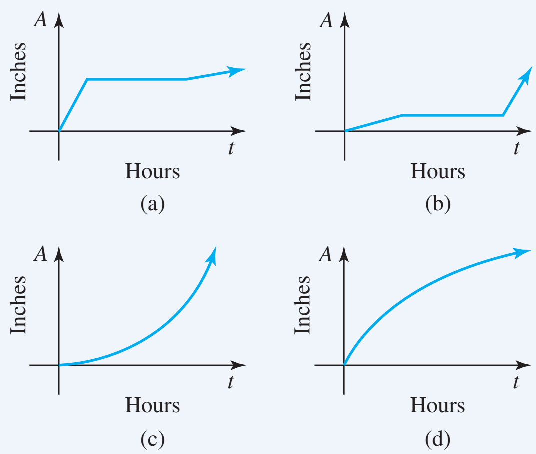

Figure 12 Exercise 2

For Exercises 3–12, identify the explanatory variable and the response variable.

3. Let T be the time (in minutes) it takes to grade N tests.
4. Let c be the total cost (in dollars) of n pencils.
5. Let F be the temperature (in degrees Fahrenheit) of an oven, and let T be the number of minutes it takes to cook a potato in the oven.
6. Let r be the rate (in gallons per hour) at which water is added to a swimming pool, and let t be the number of hours it takes to fill the pool.